B.Sc Physics Part-III

11/09/2020

Mathematical Physics

Wave equation

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2} \quad \text{---} (1)$$

We discuss here solution of the wave equation given by (1).

Let the solution of (1) is of the form

$$y = X(x)\, T(t) \quad \text{---} (2)$$

$X(x) \rightarrow$ function of x

$T(t) \rightarrow$ function of t

Substitution eq (2) in (1), we obtain

$$X \frac{d^2 T}{dt^2} = c^2 \frac{d^2 X}{dx^2} \cdot T$$

or

$$\frac{1}{X}\frac{d^2 X}{dx^2} = \frac{1}{c^2 T}\frac{d^2 T}{dt^2} \quad \text{---} (3)$$

each side should be equal to a constant say (k).

$$\frac{1}{X}\frac{d^2 X}{dx^2} = k \quad \text{and} \quad \frac{1}{c^2 T}\frac{d^2 T}{dt^2} = +k$$

or

$$\frac{d^2 X}{dx^2} - kX = 0 \quad \text{and} \quad \frac{d^2 T}{dt^2} - kc^2 T = 0$$

We obtain the following solution of above equations depending on the choices of constant k.

(i) $k = +ve$ and ~~...~~ equal to α^2.

$$X = a_1 e^{\alpha x} + a_2 e^{-\alpha x} \text{ and } T = a_3 e^{c\alpha t} + a_4 e^{-c\alpha t}$$

(ii) $k = -ve$ and equal to $-\alpha^2$

$$X = a_5 \cos\alpha x + a_6 \sin\alpha x \text{ and } T = a_7 \cos c\alpha t + a_8 \sin c\alpha t.$$

(iii) $k = 0$.

$$X = a_9 x + a_{10} \text{ and } T = a_{11} t + a_{12}.$$

Therefore, the solutions of eqn (1) are

$$y = (a_1 e^{\alpha x} + a_2 e^{-\alpha x})(a_3 e^{c\alpha t} + a_4 e^{-c\alpha t})$$

$$y = (a_5 \cos\alpha x + a_6 \sin\alpha x)(a_7 \cos c\alpha t + a_8 \sin c\alpha t)$$

$$y = (a_9 x + a_{10})(a_{11} t + a_{12})$$

From the above solution we consider that solution which is consistent with the given boundary conditions.